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**AN ANALYTICAL STUDY OF
THERMAL RADIATION IN GUN TUBES**

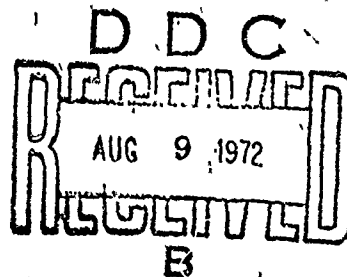


TECHNICAL REPORT

Dr. William J. Leech

NATIONAL TECHNICAL
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June 1972



RESEARCH DIRECTORATE

WEAPONS LABORATORY AT ROCK ISLAND

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U. S. ARMY WEAPONS COMMAND

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NOMENCLATURE

Symbols

C	= Specific heat
e_b	= Black body emission
f	= Friction factor
g_c	= Gravitational constant
h	= Heat transfer coefficient
K	= Thermal conductivity
k	= Absorption coefficient
k_R	= Rosseland mean absorption coefficient
L	= Length
\dot{m}	= Mass flow rate
Nu	= Nusselt number
P	= Pressure
Pr	= Prandtl number
q	= Heat flux
r	= Radial coordinate
r_o	= Outside radius
r_o^*	= Dimensionless outside radius
Re	= Reynolds number
T	= Temperature
t	= Time
u	= Velocity
u^*	= Dimensionless velocity
v	= Velocity
V_p	= Projectile velocity
x	= Spatial coordinate
y	= Spatial coordinate
y^*	= Dimensionless coordinate
β	= Thermal expansion coefficient
δ	= Boundary layer thickness
ϵ	= Eddy diffusivity, emissivity
θ	= Dimensionless temperature

λ = Wavelength
 μ = Viscosity
 ν = Viscosity
 ξ = Dimensionless parameter
 ρ = Density
 σ = Radiation parameter
 τ = Optical length
 ϕ = Rayleigh dissipation function

Subscripts

d = Based on diameter
g = Gas value
R = Radiation value
W = Wall value
X = Based on distance
 λ = Wavelength
o = Initial value
 ∞ = Free stream value

INTRODUCTION

The purpose of the present investigation is to analytically investigate the contribution of thermal radiation to gun tube heating when a hot propellant gas flows through the gun tube. Thermal radiation has been disregarded in previous gun tube heat transfer analyses. This disregard was justified on the basis of the use of the following equation:¹

$$q = \sigma \epsilon_g \epsilon_w (T_g^4 - T_w^4) \quad (1)$$

The maximum radiative heat flux was calculated by the assignment of values of unity to ϵ_g and ϵ_w . The calculated radiative flux was shown to be much less than that of experimental values. This fact was used as a justification for disregarding thermal radiation. Unfortunately, Equation 1 does not apply to the physical situation encountered in gun tubes. Equation 1 is valid for only an isothermal homogeneous gas surrounded by an isothermal surface and in the absence of simultaneous conduction or convection. The propellant gas may be characterized as optically thick. Radiation in an optically thick medium may be described as a diffusion process having a "radiation conductivity" given by²

$$k_R = \frac{16\sigma T^3}{3k} \quad (2)$$

With the use of Equation 2, under certain circumstances, the radiation conductivity in propellant gases is much greater than the thermal conductivity. Thus, the conclusion might be drawn, erroneously, that radiation is the dominant mode of heat transfer and that convection may be disregarded. Both of the approaches described are oversimplified since the essential character of energy transfer in a medium which absorbs and emits radiation is disregarded. The total heat transfer mechanism, which includes the interaction of conduction, convection, and radiation, must be considered in any rigorous analysis. The study of the interaction of radiation with conduction and convection in absorbing and emitting media is a relatively new field. The greater part of this work has been conducted since 1960. Viskanta^{3,4} studied the interaction of laminar convection and radiation in channels and tubes. Einstein⁵ considered radiant heat transfer to absorbing gases inclosed in a circular pipe with conduction, gas flow, and internal heat generation. Viskanta⁶ has also compiled an extensive review on the subject. More recently, Pearce and Emery⁷ studied the combined heat transfer by radiation and laminar convection to a radiation absorbing fluid in the entry region of a pipe. Substantially less work has been done on the interaction of radiation and turbulent flows. Nichols⁸ investigated radiation in the turbulent flow of steam in the entrance region of an annulus. Landram et al⁹ analytically investigated heat transfer in turbulent pipe flow with optically thin radiation.

The propellant gas flow in a gun tube is unsteady and turbulent. The analysis of unsteady turbulent flow is a new field and not well advanced. Heat transfer analyses for this type of flow usually involve solutions of the transient fluid dynamics equations and, then, use of some analogy to determine the heat transfer coefficient.¹⁰ No references were found in which an attempt was made to compute the unsteady turbulent temperature field. Knowledge of the temperature field is required to determine the interaction of convection with radiation since the radiant heat flux is a function of the local temperature. Thus the calculation of radiant heat flux in unsteady turbulent flow such as is found in gun tubes is beyond the present state of the art.

The approach in the present investigation is to consider a constant property, gray gas (one whose radiation properties are independent of wavelength) for steady turbulent flow in a tube. While this is an extreme simplification, the present approach should provide a substantially better method for predicting the radiative flux in gun tubes than any other previous method.

A discussion of radiation in participating media is given in the following section. The analytical section deals with the analyses of a constant property, gray, optically thick gas for steady undeveloped turbulent flow and for steady fully developed flow in a tube. The analytical results are presented and discussed.

RADIATION IN PARTICIPATING MEDIA

The expression for the conservation of energy in a moving fluid is

$$\rho C_p \frac{DT}{Dt} = - \operatorname{div} q + \beta T \frac{DP}{Dt} + \dot{q} \quad (3)$$

Equation 3 is equally applicable to both radiation participating and nonparticipating fluids. The difference is in the expression for the heat flux vector, \vec{q} . When both conduction and radiation are present, the total heat flux vector may be expressed as

$$\vec{q} = -K \operatorname{grad} T + \vec{q}_R \quad (4)$$

where $-K \operatorname{grad} T$ expresses thermal conduction and \vec{q}_R represents thermal radiation. The radiation flux vector must be formulated as a function of temperature to solve Equation 4. The heat flux vector can be an extremely complicated mathematical function. This vector is dependent upon the flow field, the gas properties, and the geometry of the surrounding inclosure. To reduce the heat flux vector to a mathematically tractable form usually requires a number of simplification assumptions.

One of the most important parameters by which radiation is described in a participating medium is the optical thickness, τ_λ .

$$\tau_\lambda = k_\lambda L = \frac{L}{\lambda_p} \quad (5)$$

The optical thickness represents the ratio of a characteristic dimension of the system to the photon mean free path length. The value of the optical distance characterizes three separate radiation regimes. When the value of τ_λ is much less than one, the radiation is called optically thin. The photon mean free path is much greater than the system dimensions. Any photon emitted by a gas volume will travel directly to the bounding surfaces without interacting with any other gas volume. Thus, the fluid absorbs only radiation emitted by the boundary surfaces. This is also referred to as negligible self-absorption. When τ_λ is approximately equal to one, radiation exchange occurs between all the fluid elements, and between all fluid elements and the boundaries. This results in an integral expression for the radiation flux vector.

The third regime involves situations where τ_λ is much greater than one. The photon mean free path length is much less than the system dimensions and the radiation may be characterized as a photon continuum. Radiation transfer within the medium becomes a diffusion process. As shown by Sparrow and Cess,² the heat flux vector for this regime is given by

$$\vec{q}_{R\lambda} = - \frac{4}{3k_\lambda} \text{grad } e_{b\lambda} \quad (6)$$

Equation 6 is valid within one photon mean free path length from the bounding surface. Beyond a distance of one mean free path length, no photons emitted by the surface will be present. Thus, radiation from the boundary will not be detected by the medium, and the heat transfer within the medium will be independent of the surface emittance. Some question exists as to whether Equation 6 applies at the boundary itself. However, Sparrow and Cess argue that if Equation 6 were not valid at the boundary, a discontinuity in radiation flux would occur, which is physically impossible. They conclude that Equation 6 is valid at the boundaries and throughout the medium when optically thick conditions prevail.

Equation 6 gives the heat flux vector as a function of wavelength. The assumption of a gray gas eliminates the dependence on wavelength. The total radiation heat flux is determined by integration over all wavelengths.

$$q_R = - \int_0^\infty \frac{4}{3k_\lambda} \frac{de_{b\lambda}}{d\lambda} d\lambda \quad (7)$$

Integration of Equation 7 results in

$$q_R = \frac{4}{3k_R} \frac{de_b}{dy} = \frac{16\sigma T^3}{3k_R} \frac{dT}{dy} \quad (8)$$

where

$$\frac{1}{k_R} = \int_0^\infty \frac{1}{k_\lambda} \frac{de_{b\lambda}}{de_b} d\lambda \quad (9)$$

The quantity k_R is called the Rosseland mean absorption coefficient, and the quantity $16\sigma T^3/3k_R$ is the radiation conductivity

The absorption coefficient, k_λ , must be known to evaluate Equation 9. The function k_λ has been experimentally determined for a number of pure gases at room temperature and at atmospheric pressure. The absorption or emission of radiation by a gas occurs at fixed frequencies over narrow frequency bands. The emission and absorption at these frequencies are due to transitions between energy levels of the atoms and molecules that constitute the gas. The frequency bands may be collision broadened. This phenomenon occurs when the pressure and temperature become high, and the frequency of molecular and atomic collisions is increased. In mixtures of gases, an overlapping of absorption bands may exist. While k_λ may, in theory, be computed from quantum mechanics, experimental data are usually necessary. Unfortunately, the locating of any experimental data for the absorption coefficient as a function of temperature and pressure, and for the currently used propellant gases was impossible. The primary radiation participating constituents of propellant gases are carbon monoxide, 54 percent by weight; carbon dioxide, 17 percent by weight; and water vapor, 15 percent by weight. Tien¹² has used experimental data to compute the Rosseland mean absorption coefficient for each of these gases as functions of temperature and pressure. To simply add the absorption coefficient of each constituent gas to find the total absorption coefficient is impossible since some of the absorption bands overlap. However, to obtain the minimum value of absorption coefficient is possible since it is at least as large as the largest absorption coefficient of any of the constituent gases. This minimum mean absorption coefficient can be combined with some characteristic length to determine a characteristic optical dimension. A parametric study can then be performed by variation of the optical dimension. The results will show the range of optical distances for which radiation in gun tubes is important. Thus, rational judgments can be made about the contribution of radiation to overall gun tube heating.

The approximate minimum value of k_R may be determined from Figures 22, 23, and 24 of Tiens article, and is

$$k_{R_{\min}} \approx \frac{60}{\text{atm ft}} \quad (10)$$

For a gun tube pressure of 30,000 psi and a bore diameter of 0.3 inch, the optical diameter has a value of 3000. This value is much greater than one, and the radiation is in the optically thick regime.

THEORETICAL ANALYSIS

The analysis of radiation-convection interaction for optically thick turbulent flow in a tube is outlined in this section. The assumption is that fluid properties are constant; therefore, the fluid flow is independent of the temperature distribution. Additional assumptions are that the flow is steady and that viscous dissipation is negligible. The flow of both undeveloped and fully developed turbulence is considered. The various flow regimes for turbulent flow in a tube are illustrated in Figure 1.

Undeveloped Turbulent Flow

When the thickness of the hydrodynamic boundary layer is much less than the tube radius, the flow is not fully developed and is closely approximated by the flow of a semi-infinite fluid over a flat plate. The thickness of the thermal boundary layer is nearly the same as the thickness of the hydrodynamic boundary layer if the Prandtl number, $Pr = \nu/\alpha$, is close to unity. This condition is true for most gases. The equality of the boundary layer thicknesses will be assumed in this analysis. Equation 3, subject to all the restrictions mentioned, may be written as

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[\left(\frac{k}{\rho c} + \frac{16\sigma T^3}{3k_R \rho c} + \epsilon \right) \frac{\partial T}{\partial y} \right] \quad (11)$$

where ϵ is the turbulent eddy diffusivity for heat transfer. The case to be considered is that in which the tube walls are at a constant temperature, T_0 , and the inviscid core is at a constant temperature, T_∞ . A three-layer model will be used to describe the turbulent flow. The three layers are the laminar sublayer, buffer layer, and turbulent layer.¹³ In the two sublayers near the wall, the radial velocity, v , is approximately equal to zero. Since the tube surface temperature is constant, $\partial T/\partial x$ is also approximately equal to zero in the sublayers.

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Thus, for the sublayers, Equation 11 reduces to

$$\frac{dT}{dy} \left[\left(\frac{K}{\rho c} + \frac{16\sigma T^3}{3k_R \rho c} + \epsilon \right) \right] = 0 \quad (12)$$

Equation 12 may be integrated once to give

$$\left(\frac{K}{\rho c} + \frac{16\sigma T^3}{3k_R \rho c} + \epsilon \right) \frac{dT}{dy} = c \quad (13)$$

When $y = 0$, $\epsilon = 0$ and

$$\left(\frac{K}{\rho c} + \frac{16\sigma T_o^3}{3k_R \rho c} \right) \frac{dT}{dy} \Big|_0 = -q_o \quad (14)$$

where $q > 0$ indicates that heat is being added to the fluid. Equation 14 is used to find the constant of integration and Equation 13 may be rearranged to give

$$\frac{dT}{dy} = \frac{-q_o}{\rho c \left(\frac{K}{\rho c} + \frac{16\sigma T^3}{3k_R \rho c} + \epsilon \right)} \quad (15)$$

A different procedure is used to determine the temperature distribution in the turbulent layer. Reynolds' analogy is used in the turbulent layer. The relation between the velocity and temperature profiles is

$$\frac{dT}{du} = \frac{-q_o}{g_c r_o c} \quad (16)$$

Equation 16 is valid only if the total diffusivities for heat and momentum are equal and if the turbulent diffusivities are much greater than the molecular diffusivities. These conditions are usually satisfied in the turbulent flow of gases when no radiation is present. Some care must be exercised in the use of Equation 16 for the present investigation. The total thermal conductivity is the sum of the molecular conductivity and the radiation conductivity. If the radiation conductivity is much higher than the molecular conductivity, the Prandtl number calculated from the total conductivity may become quite small. Molecular diffusion may be of the same order of magnitude as turbulent diffusion

in this case. Equation 16 will be valid only if the quantity ε/ν is much larger than the quantity $(1 + 16\sigma T^3/3k_R K)/Pr$ throughout the turbulent layer. This requirement may be difficult to satisfy near the edge of the buffer layer.

Experimental data for the velocity profiles in steady turbulent flow have been correlated so that a dimensionless velocity, u^* , is a known function of a dimensionless coordinate, y^* . This function is referred to as the universal velocity profile for turbulent flow. To utilize these data, the governing equations must be expressed in terms of these dimensionless variables, which are defined as

$$u^* = \frac{u}{\sqrt{g_c \tau_o / \rho}} \quad (17)$$

and

$$y^* = y \frac{\sqrt{g_c \tau_o / \rho}}{\nu} \quad (18)$$

Also, the expressing of temperature in a dimensionless form is desirable so that all significant parametric groups affecting the temperature distribution can be isolated. The following dimensionless temperature is defined:

$$\theta = T \frac{\rho c u_\infty}{q_o} = T \left(\frac{K}{q_o x} \right) Pr Re_x \quad (19)$$

The introduction of the dimensionless variables into the differential equation for the sublayers gives

$$\frac{d\theta}{dy^*} = \frac{-1}{\left(\frac{fx}{2}\right)^{1/2} \left(\frac{1}{Pr} + \frac{\varepsilon_x \theta^3}{Pr^4 Re_x^3} + \frac{\varepsilon}{\nu}\right)} \quad (20)$$

where

$$\varepsilon_x = \frac{16}{3} \left(\frac{\sigma \nu}{K u_\infty}\right) \left(\frac{q_x}{K}\right)^3 \left(\frac{u_\infty}{k_R \nu}\right) \quad (21)$$

The friction factor for steady turbulent flow over a flat plate is¹³

$$f_x = 0.059 \text{ Re}_x^{-0.2} \quad (22)$$

Thus, the governing differential equation for the sublayers becomes

$$\frac{d\theta}{dy^*} = \frac{-5.822 \text{ Re}_x^{0.1}}{\left[\frac{1}{\text{Pr}} + \frac{\epsilon_x \theta^3}{\text{Pr}^4 \text{Re}_x^3} + \frac{\epsilon}{v} \right]} \quad (23)$$

The dimensionless form of the differential equation for the turbulent layer is

$$\frac{d\theta}{du^*} = -5.822 \text{ Re}_x^{0.1} \quad (24)$$

Several different functional forms for the universal velocity profile and eddy diffusivity have been proposed. Kays¹³ recommends the following relations:

Laminar Sublayer $0 < y^* < 5$

$$u^* = y^*, \quad \frac{\epsilon}{v} = 0 \quad (25)$$

Buffer Layer $5 < y^* < 30$

$$u^* = -3.05 + 5 \ln y^*, \quad \frac{\epsilon}{v} = \frac{y^*}{5} - 1 \quad (26)$$

Equation 24 may be integrated directly to give

$$\theta = 5.822 u^* \text{ Re}_x^{-0.1} + c \quad (27)$$

The constant of integration is determined by evaluation of Equation 27 at $y^* = 30$ where, from Equation 26, $u^* = 14$. The dimensionless temperature distribution in the turbulent layer is given by

$$\theta(y^*) = \theta(30) - 5.822\text{Re}_x^{0.1} (u^* - 14) \quad (28)$$

When Equation 28 is evaluated in the free stream, $y^* = \infty$ and $U^*_{\infty} = 1/\sqrt{f_x/2}$, the following result is obtained:

$$\theta(\infty) - \theta(30) = - 5.822\text{Re}_x^{0.1} (5.822\text{Re}_x^{0.1} - 14) \quad (29)$$

The set of equations from which the total temperature difference across the turbulent boundary layer can be obtained are

Laminar Sublayer, $0 < y^* < 5$

$$\frac{d\theta}{dy^*} = - 5.822\text{Re}_x^{0.1} \left[\frac{\text{Pr}}{\xi_x \theta^3} \right] \left[1 + \frac{\xi_x \theta^3}{(\text{PrRe}_x)^3} \right]$$

Buffer Layer, $5 < y^* < 30$

$$\frac{d\theta}{dy^*} = \frac{- 5.822\text{Re}_x^{0.1}}{\left[\frac{1}{\text{Pr}} + \frac{\xi_x \theta^3}{\text{Pr}^4 \text{Re}_x^3} + \frac{y^*}{5} - 1 \right]} \quad (30)$$

Turbulent Layer, $30 < y^* < \infty$

$$\theta(\infty) - \theta(30) = - 5.822\text{Re}_x^{0.1} (5.822\text{Re}_x^{0.1} - 14)$$

The heat transfer coefficient is defined as

$$h = \frac{q_o}{(T_{\infty} - T_o)} \quad (31)$$

and the Nusselt number is defined as

$$\text{Nu}_x = \frac{hx}{K} = \frac{q_o x}{K(T_{\infty} - T_o)} \quad (32)$$

When the dimensionless variables are introduced into Equation 32, the following expression for the Nusselt number is obtained:

$$Nu_x = \frac{Pr Re_x}{\theta(\infty) - \theta(0)} \quad (33)$$

The set of governing equations for the temperature profile may be solved numerically by the fourth order Runge-Kutta method.¹⁴ The Nusselt number is then evaluated from Equation 33. The parameter ξ_x may be varied to give the Nusselt number as functions of the Reynolds number and ξ_x . These results will apply for the case where the heat flux at the tube wall is specified. A different procedure of solution is required when the wall temperature and free stream temperature are specified. The given wall temperature is specified as the initial value. The desired values are assigned to Re_x , Pr , $(\sigma v/Ku_\infty)$, and $(u_\infty/k\rho v)$. The value of $(q_0 x/K)$ is adjusted, through an iterative procedure, until a solution with the correct free stream temperature is obtained. The results give the Nusselt number as a function of the Reynolds number, the Prandtl number, wall temperature, free stream temperature, thermal conductivity, and $(u_\infty/k\rho v)$. The results of a numerical evaluation are given in the following section.

Fully Developed Turbulent Flow

The flow in a tube is fully developed, in the hydrodynamic sense, when the thickness of the hydrodynamic boundary layer is equal to the tube radius. The velocity profile in the tube is then independent of axial location, and the radial velocity component is zero. However, this does not imply that the flow is fully developed in the thermal sense since the fluid temperature will still be a function of the axial location. The temperature gradient in the axial direction will be a constant if the heat flux at the tube wall is a constant. Equation 3, subject to all previous assumptions and the additional assumptions of fully developed flow and constant heat flux, is

$$u \frac{dT}{dx} = \frac{1}{r} \frac{d}{dr} \left[r \left(\frac{K}{\rho c} + \frac{16\sigma}{3k_p \rho c} + \epsilon \right) \frac{dT}{dr} \right] \quad (34)$$

The universal velocity profile is expressed in terms of the distance from the tube wall. Thus, a different coordinate system, which originates at the wall, must be defined. The new coordinate is

$$y = r_0 - r \quad (35)$$

Equation 34, written in terms of the new coordinate, is

$$u \frac{dT}{dx} = \frac{1}{(r_0 - y)} \frac{d}{dy} \left[(r_0 - y) \left(\alpha + \frac{16\sigma T^3}{3k_p \rho c} + \epsilon \right) \frac{dT}{dy} \right] \quad (36)$$

The temperature gradient in the axial direction can be determined by performance of an energy balance on a volume of gas in the tube. The temperature gradient is expressed in terms of the constant wall heat flux.

$$\frac{dT}{dx} = \frac{-2q_0}{r_0 \rho c v} \quad (37)$$

where $q_0 > 0$ indicates that heat is transferred from the fluid to the wall.

Equation 36 may be integrated once with respect to y . The constant of integration is determined from the condition that the radial temperature gradient is zero at the tube centerline. The results, when combined with Equation 37, are

$$\left[(r_0 - y) \left(\alpha + \frac{16\sigma T^3}{3k_p \rho c} + \epsilon \right) \right] \frac{dT}{dy} = \frac{2q_0}{r_0 \rho c v} \left[\int_0^{r_0} u(r_0 - y) dy - \int_0^y u(r_0 - y) dy \right] \quad (38)$$

The dimensionless variables, Equations 17, 18, and 19, are now introduced into Equation 38. The dimensionless form of the energy equation becomes

$$(r_0^* - y^*) \left(\frac{1}{Pr} + \frac{\epsilon_d \theta^3}{Pr^4 Re_d^3} + \frac{\epsilon}{v} \right) \frac{d\theta}{dy^*} = \frac{4}{Re_d \left(\frac{f}{L} \right)^{1/2}} \left[\int_0^{r_0^*} u^*(r_0^* - y^*) dy^* - \int_0^{y^*} u^*(r_0^* - y^*) dy^* \right] \quad (39)$$

where

$$\epsilon_d = \frac{16}{3} \left(\frac{\sigma d}{K} \right) \left(\frac{q_0 d}{K} \right)^3 \frac{1}{\tau_d} \quad (40)$$

Kays¹³ gives the following values for the friction factors for fully developed turbulent flow in a smooth tube:

$$f = 0.079 \text{ Re}_d^{-0.25}, \quad 5000 < \text{Re}_d < 30,000 \quad (41)$$

and

$$f = 0.046 \text{ Re}_d^{-0.2}, \quad 30,000 < \text{Re}_d < 1,000,000 \quad (42)$$

Either Equation 41 or 42 may be substituted into Equation 39, dependent upon the Reynolds number of interest.

The relations for u^* and ϵ/ν for the laminar sublayer and the buffer layer are again given by Equations 25 and 26. The following values were used for the turbulent layer, $30 < y^* < r_o^*$:

$$u^* = 5.5 + 2.5 \ln y^*, \quad \frac{\epsilon}{\nu} = \frac{y^*}{2.5} \left(1 - \frac{y^*}{r_o^*} \right) \quad (43)$$

The expressions for the dimensionless velocity, u^* , and ϵ/ν are substituted into the integrals on the right side of Equation 39. When the integrals are evaluated, algebraic functions of r_o^* and y^* are obtained. The differential equations for the layers are found as

$$\text{Laminar Sublayer, } \frac{d\theta}{dy^*} = F_1(\theta, y^*)$$

$$\text{Buffer Layer, } \frac{d\theta}{dy^*} = F_2(\theta, y^*)$$

$$\text{Turbulent Layer, } \frac{d\theta}{dy^*} = F_3(\theta, y^*) \quad (44)$$

where

$$F_1(\theta, y^*) = \left\{ \frac{4}{\text{Re}_d \left(\frac{f}{2}\right)^{\frac{1}{2}} (r_0^* - y^*) \left(\frac{1}{\text{Pr}} + \frac{\xi_d \theta^3}{\text{Pr}^4 \text{Re}_d^3}\right)} \right\} \times$$

$$\left\{ I_1(r_0^*) + I_2(r_0^*) + I_3(r_0^*) - y^{*2} \left(\frac{r_0^*}{2} - \frac{y^*}{3} \right) \right\} \quad (45)$$

$$F_2(\theta, y^*) = \left\{ \frac{4}{\text{Re}_d \left(\frac{f}{2}\right)^{\frac{1}{2}} (r_0^* - y^*) \left(\frac{1}{\text{Pr}} + \frac{\xi_d \theta^3}{\text{Pr}^4 \text{Re}_d^3} + \frac{y^*}{5} - 1\right)} \right\} \times$$

$$\left\{ I_2(r_0^*) + I_3(r_0^*) + 3.05 (r_0^* y^* - \frac{y^{*2}}{2} - 5r_0^* + 12.5) \right.$$

$$\left. - 5r_0^* (y^* \ln y^* - y^* - 5 \ln 5 + 5) \right.$$

$$\left. + 5 \left(\frac{y^{*2}}{2} \ln y^* - \frac{y^{*2}}{4} - 12.5 \ln 5 + 6.25 \right) \right\} \quad (46)$$

$$F_3(\theta, y^*) = \left\{ \frac{4}{\text{Re}_d \left(\frac{f}{2}\right)^{\frac{1}{2}} (r_0^* - y^*) \left[\frac{1}{\text{Pr}} + \frac{\xi_d \theta^3}{\text{Pr}^4 \text{Re}_d^3} + \frac{y^*}{2.5} \left(1 - \frac{y^*}{r_0^*} \right) \right]} \right\} \times$$

$$\left\{ I_3(r_0^*) - 5.5 (r_0^* y^* - \frac{y^{*2}}{2} - 30r_0^* + 450) \right.$$

$$\left. - 2.5 [r_0^* (y^* \ln y^* - y^* - 30 \ln 30 + 30)] \right.$$

$$\left. + 2.5 \left(\frac{y^{*2}}{2} \ln y^* - \frac{y^{*2}}{4} - 450 \ln 30 + 225 \right) \right\} \quad (47)$$

and

$$I_1(r_0^*) = \int_0^{r_0^*} u^*(r_0^* - y^*) dy^* = 25 \left(\frac{r_0^{*2}}{2} - \frac{5}{3} \right) \quad (48)$$

$$\begin{aligned}
I_2(r_o^*) &= \int_5^{30} u^*(r_o^* - y^*) dy^* = -3.05 (25r_o^* - 437.5) \\
&\quad + 5r_o^*(30 \ln 30 - 5 \ln 5 - 25) \\
&\quad - 5(450 \ln 30 - 12.5 \ln 5 - 218.75) \quad (49)
\end{aligned}$$

$$\begin{aligned}
I_3(r_o^*) &= \int_{30}^{r_o^*} u^*(r_o^* - y^*) dy^* = 5.5 \left(\frac{r_o^{*2}}{2} - 30r_o^* + 450 \right) \\
&\quad + 2.5 [r_o^*(r_o^* \ln r_o^* - 30 \ln 30 - r_o^* + 30)] \\
&\quad - 2.5 \left[\frac{r_o^{*2}}{2} \ln r_o^* - \frac{r_o^{*2}}{4} - 450 \ln 30 + 225 \right] \quad (50)
\end{aligned}$$

The complete temperature distribution in the tube can be determined from the solution of these equations.

The value of r_o^* , which is a function of the Reynolds number, must be known before the differential equations can be solved. The mass flow rate in the tube is

$$\dot{m} = 2\pi\rho \int_0^{r_o} u r dr \quad (51)$$

The Reynolds number may be defined in terms of the mass flow rate as

$$Re_d = \frac{2\dot{m}}{\pi\rho\nu r_o} \quad (52)$$

When Equations 51 and 52 are combined and expressed in dimensionless form, the following expression may be obtained for r_o^* :

$$r_o^* = \frac{4}{Re_d} \int_0^{r_o^*} u^*(r_o^* - y^*) dy^* \quad (53)$$

or

$$r_o^* = \frac{4}{Re_d} [I_1(r_o^*) + I_2(r_o^*) + I_3(r_o^*)] \quad (54)$$

Equation 54 may be solved numerically for r_o^* by the Newton-Raphson method.¹⁴

The Nusselt number can be calculated when the complete temperature distribution is known. The Nusselt number is defined as

$$Nu = \frac{hd}{K} = \frac{q_o d}{K(T_\infty - T_o)} \quad (55)$$

The difference between the wall temperature and the centerline temperature, or the difference between the wall and bulk temperature, may be used to evaluate Equation 55. Introduction of the dimensionless variables into Equation 55 gives the Nusselt number on the basis of maximum temperature difference.

$$Nu_c = \frac{Re_d Pr}{(\theta_\infty - \theta_o)} \quad (56)$$

which can be evaluated when the dimensionless centerline temperature has been calculated.

The bulk temperature is defined as

$$T_b = \frac{2}{r_o^2 V} \int_0^{r_o} u T r dr \quad (57)$$

The dimensionless variables are introduced into Equation 57 which is then substituted into the dimensionless form of Equation 55. The following expression is obtained for the Nusselt number on the basis of the bulk temperature difference.

$$Nu_b = \frac{Re_d Pr r_o^{*2}}{\sqrt{2f} \int_0^{r_o^*} u^* \theta(r_o^* - y^*) dy^* - r_o^{*2} \theta_w} \quad (58)$$

Equation 58 can be numerically evaluated when the temperature distribution is known.

The governing equations were evaluated for the case where ξ_d is specified, and for the case where the wall and centerline temperature were specified. An iterative solution was required in the latter case. The analytical results are presented in the following section.

ANALYTICAL RESULTS

The differential equations, derived in the previous section, were numerically evaluated by use of the fourth order Runge-Kutta algorithm. The results for turbulent flow over a flat surface are shown in Figures 2, 3, and 4. The Nusselt number is plotted against the Reynolds number as a function of ξ_x and the dimensionless wall temperature in Figure 2. Radiation effects are important if Re_x is less than 2×10^5 . For larger values of the Reynolds number, turbulent convection is the dominant mode of heat transfer. The relative importance of radiation heat transfer is strongly dependent on the value of ξ_x . Radiation is important when ξ_x is large. The value of ξ_x is large when the temperature is high or the absorption coefficient is small. The value of the radiation contribution also depends on the value of the bore surface temperature. A higher surface temperature results in a relatively larger heat transfer contribution by radiation. The Nusselt number is plotted against the Reynolds number, in Figures 3 and 4, as a function of centerline temperature and the parameter u_∞/ν . A bore surface temperature of $1000^\circ R$ is shown in Figure 3, and a bore surface temperature of $2000^\circ R$ is shown in Figure 4. The radiation contribution is important when Re_x is less than 2×10^5 . Radiation is significant at the higher Reynolds numbers only if the parameter u_∞/ν is large. This again means that the absorption coefficient must be small. The values of u_∞ and ν are known from the flow conditions in the gun tubes. Thus, the relative importance of radiation can be quickly estimated when the absorption coefficient is known. The radiation contribution for a given centerline temperature is greater when the surface temperature is higher as shown in Figures 3 and 4. This means that, as the barrel temperature rises, the heat flux does not decrease as much as would be calculated by the convective heat transfer theory.

The Nusselt number for fully developed turbulent flow is plotted in Figures 5, 6, and 7. The Nusselt number is plotted against the Reynolds number as a function of ξ_x and the dimensionless wall temperature. Radiation is important when the Reynolds number is less than 2×10^5 . Turbulent convection heat transfer is dominant for larger Reynolds numbers. The relative importance of radiation is strongly dependent upon the value of ξ_d . Radiation is important for large values of ξ_d , which implies that the temperature must be high or the absorption coefficient small. Higher bore surface temperatures result in a relatively higher contribution of radiation to the total heat transfer. The Nusselt number is plotted, against the Reynolds number

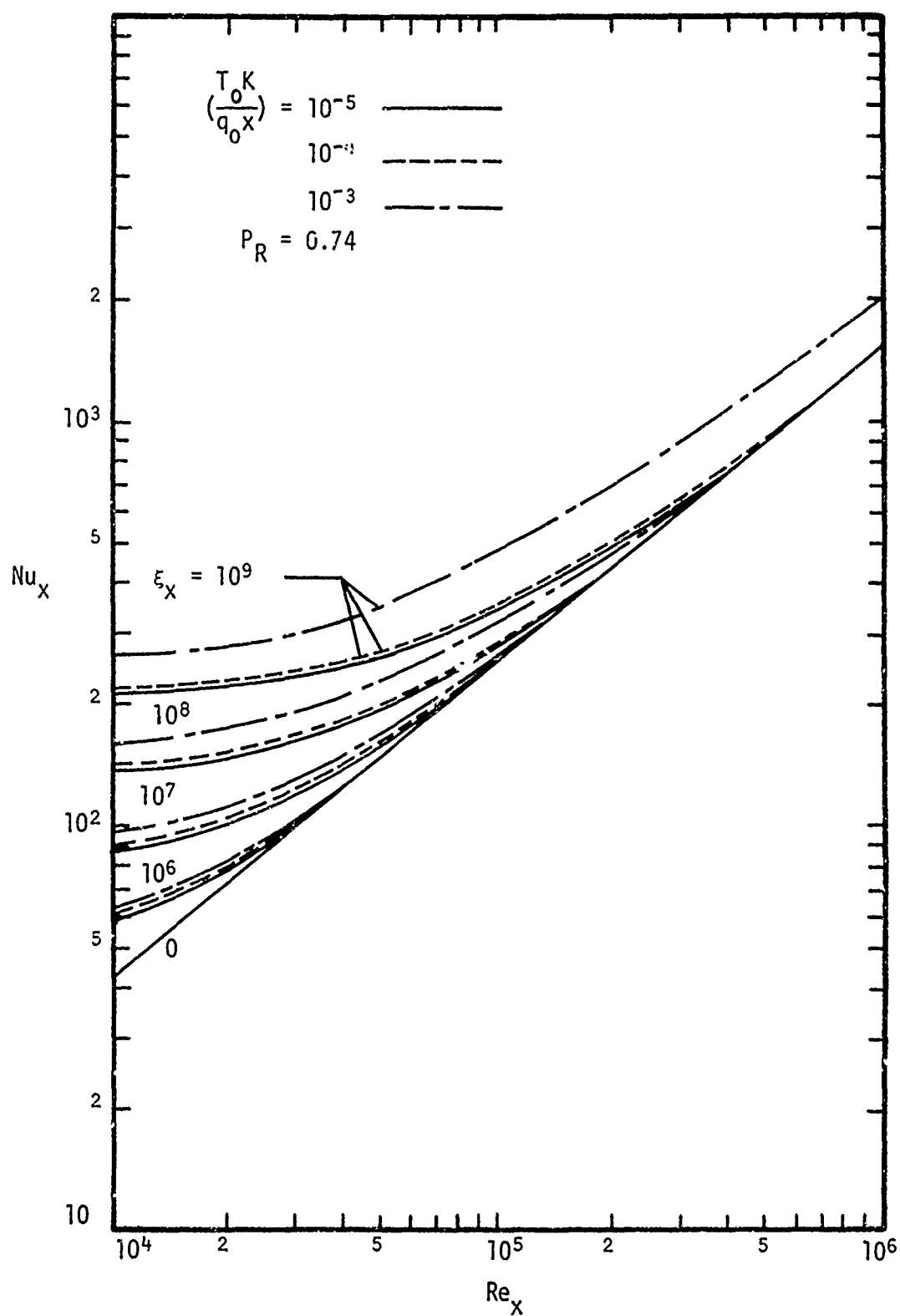


FIGURE 2. NUSSELT NUMBER VERSUS REYNOLDS NUMBER AS A FUNCTION OF ϵ_x AND $(T_0 K / q_0 x)$ FOR TURBULENT FLOW OVER A FLAT PLATE WITH OPTICALLY THICK RADIATION

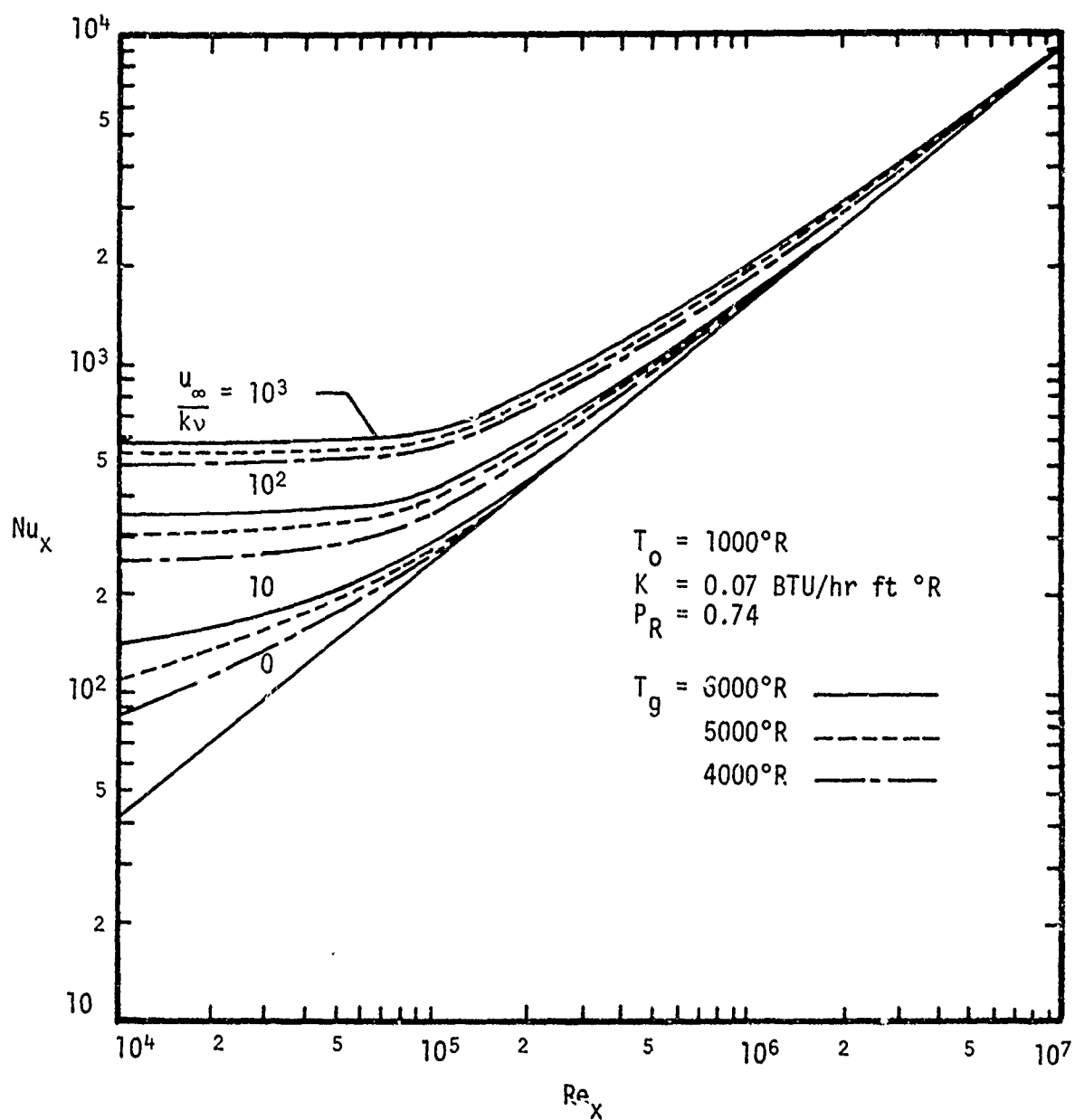


FIGURE 3. NUSSELT NUMBER VERSUS REYNOLDS NUMBER AS A FUNCTION OF T_g AND u_∞/kv FOR TURBULENT FLOW OVER A FLAT PLATE WITH OPTICALLY THICK RADIATION

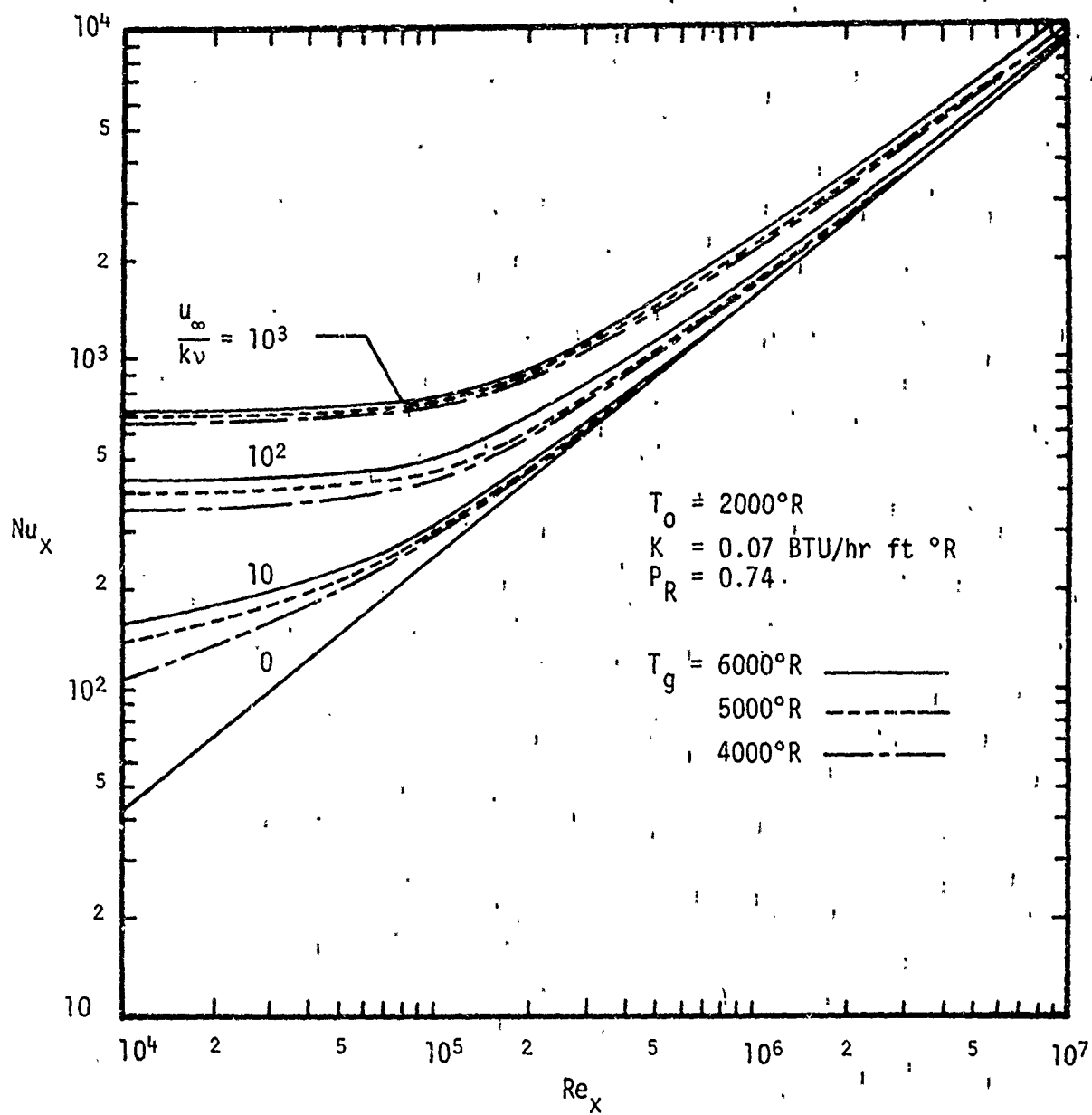


FIGURE 4. NUSSELT NUMBER VERSUS REYNOLDS' NUMBER AS A FUNCTION OF T_g AND u_∞/kv FOR TURBULENT FLOW OVER A FLAT PLATE WITH OPTICALLY THICK RADIATION

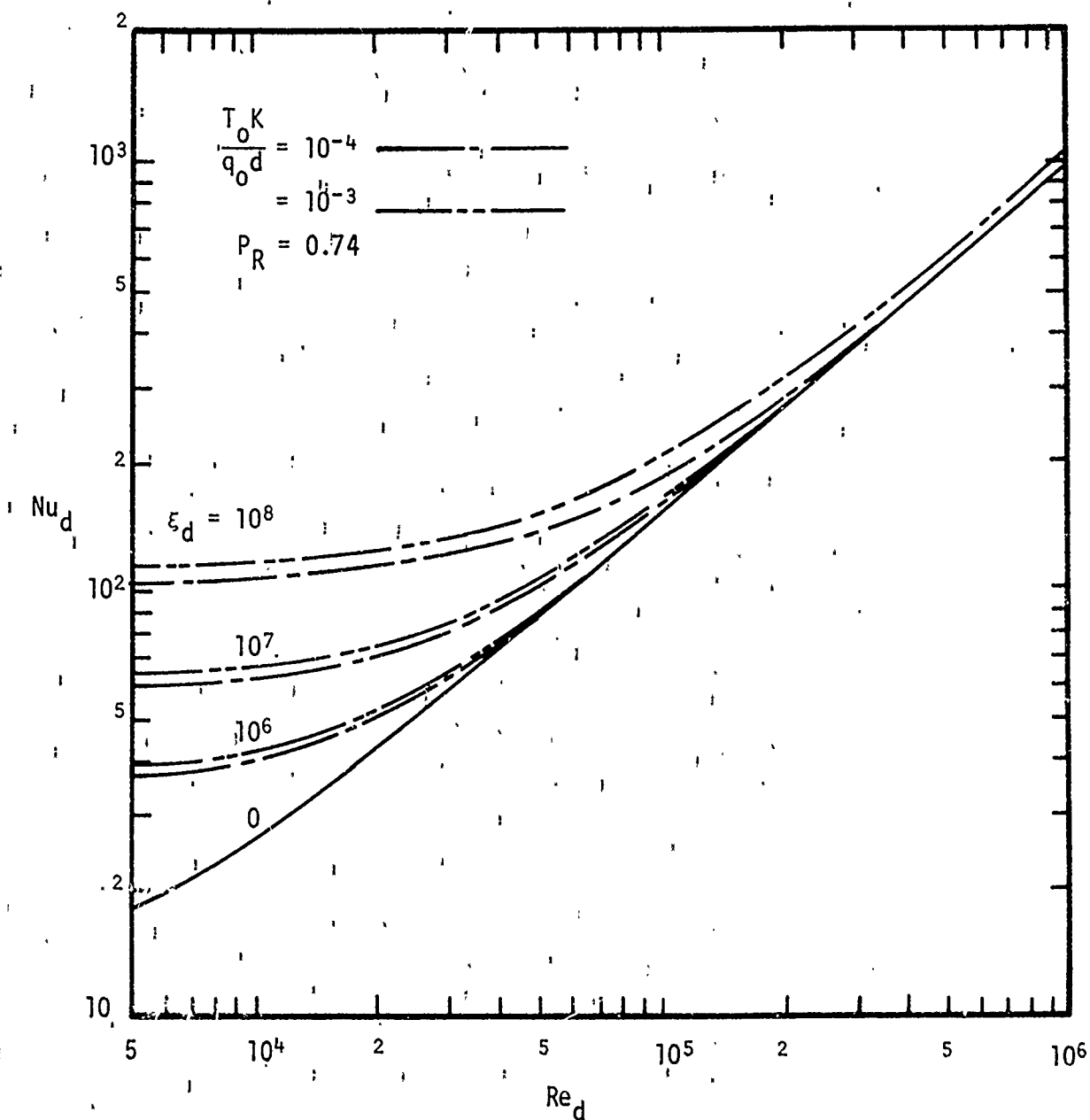


FIGURE 5. NUSSELT NUMBER, BASED ON CENTERLINE TEMPERATURE DIFFERENCE, VERSUS REYNOLDS NUMBER AS A FUNCTION OF ξ_d AND $(T_o K/q_o d)$ FOR FULLY DEVELOPED TURBULENT FLOW IN A TUBE WITH OPTICALLY THICK RADIATION

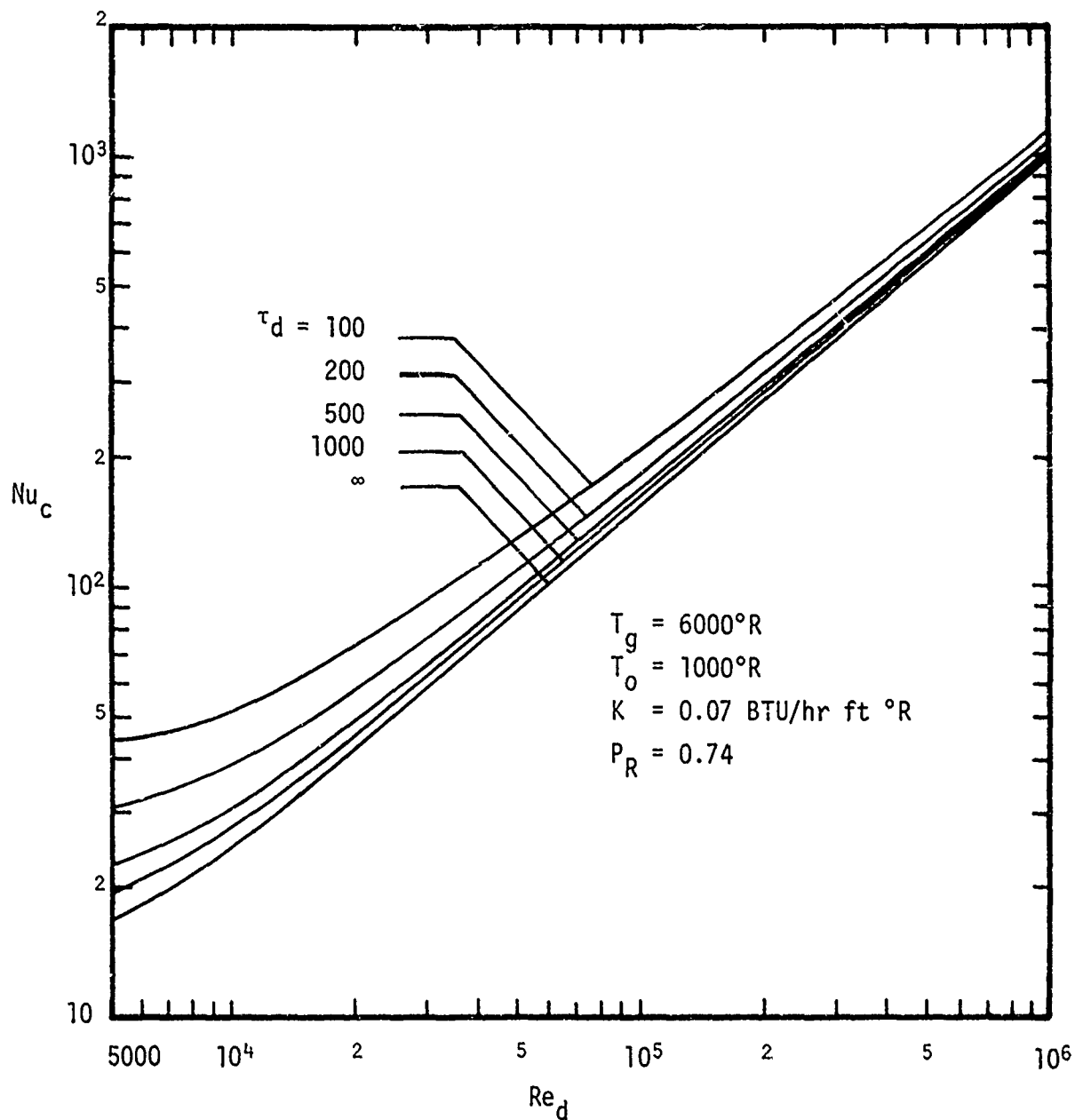


FIGURE 6. NUSSELT NUMBER, BASED ON CENTERLINE TEMPERATURE DIFFERENCE, VERSUS REYNOLDS NUMBER AS A FUNCTION OF OPTICAL DIAMETER FOR OPTICALLY THICK, FULLY DEVELOPED, TURBULENT FLOW IN A TUBE. $T_g = 6000^\circ\text{R}$, $T_o = 1000^\circ\text{R}$

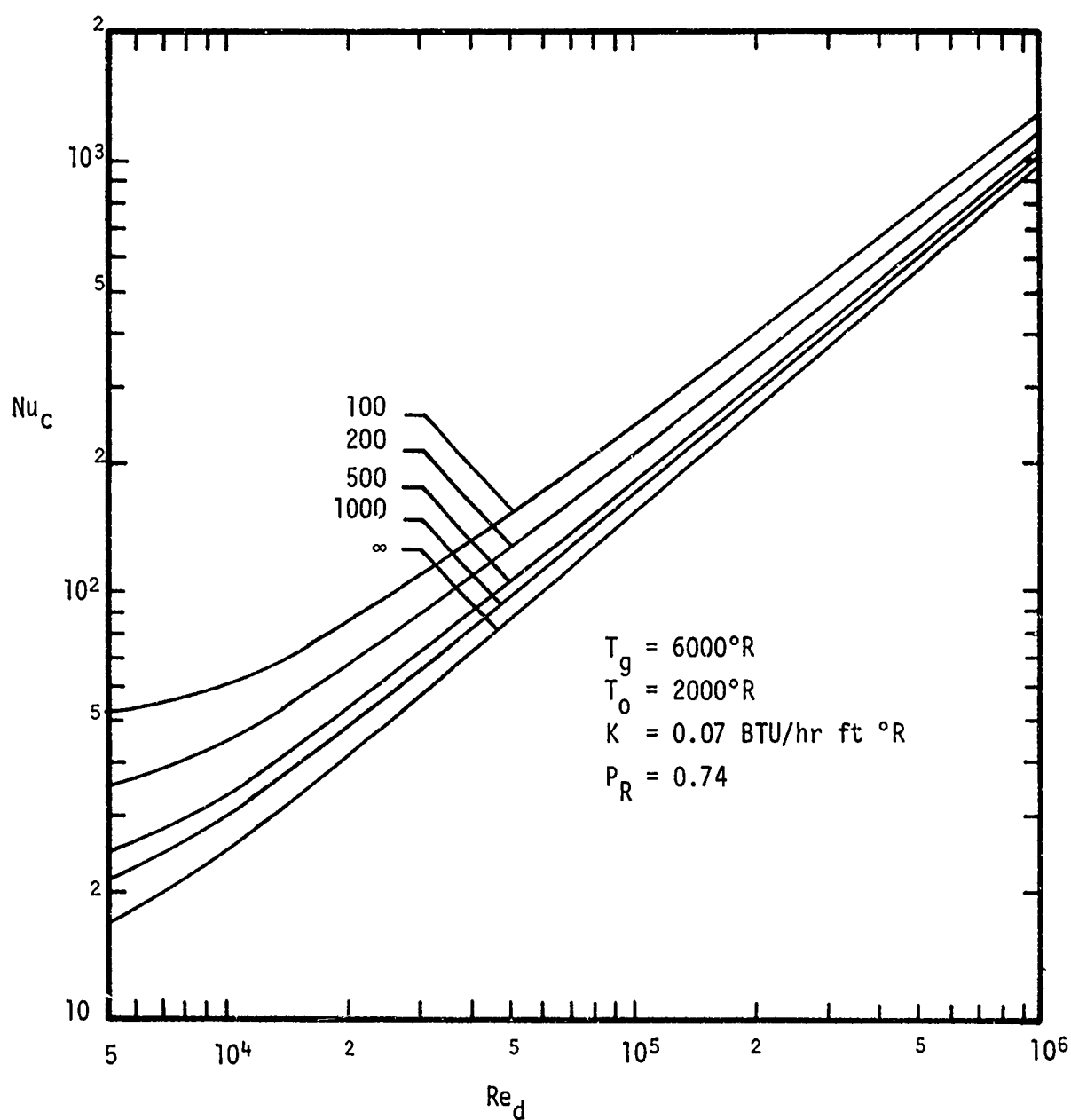


FIGURE 7. NUSSELT NUMBER, BASED ON CENTERLINE TEMPERATURE DIFFERENCE, VERSUS REYNOLDS NUMBER, AS A FUNCTION OF OPTICAL DIAMETER FOR OPTICALLY THICK, FULLY DEVELOPED TURBULENT FLOW IN A TUBE. $T_g = 6000^\circ R$, $T_o = 2000^\circ R$

as a function of the optical diameter, as shown in Figures 6 and 7. The centerline temperature is 6000°R in both of these figures: the surface temperature is 1000°R in Figure 6, and 2000°R in Figure 7. The contribution of radiation is a strong function of the optical diameter. For large values of the optical diameter ($\tau_d > 1000$), the effects of radiation are small. This occurs because the major portion of the radiation emitted by the gas is absorbed before the radiation strikes the bore surface. The effects of radiation are greater for higher wall temperatures.

The data presented in the figures may be used to quickly estimate the importance of radiation heat transfer in any gun tube in which the flow conditions are known. The XM140 gun tube may be considered as an example. The projectile velocity, gas density, pressure, and gas temperature have been plotted against projectile displacement by Dahm and Anderson.¹⁰ When the base of the projectile is three inches from the breech end, the conditions in the gun tube are

$$\begin{aligned} L &= 3 \text{ in} \\ P &= 25,000 \text{ lb/in}^2 \\ V_p &= 325 \text{ ft/sec} \\ \rho &= 7 \text{ lb/ft}^3 \\ T &= 5000^\circ\text{R} \end{aligned}$$

The value of the viscosity can be calculated from the relation¹⁰

$$\mu = 0.044 \left(\frac{T}{530} \right)^{0.74} \frac{\text{lb}}{\text{hr ft}} \quad (59)$$

The value of the Rosseland mean absorption coefficient can be estimated from Equation 10. These data were used to calculate Re_x and $u_\infty/k_R v$ as functions of x . The calculated values are listed below:

$x \text{ (in)}$	Re_x	$u_\infty/k_R v$
0	0	0
0.5	2.34×10^5	0.55×10^2
1.0	9.36×10^5	1.10×10^2
1.5	2.11×10^6	1.65×10^2
2.0	3.75×10^6	2.20×10^2
2.5	5.85×10^6	2.75×10^2
3.0	8.43×10^6	3.30×10^2

The boundary layer thickness can be calculated from¹³

$$\delta = 0.3814 \times \text{Re}_x^{-0.2} \quad (60)$$

The maximum boundary layer thickness is 0.047 inch which is much less than that of the tube radius, 0.59 inch. The flow is not fully developed, and the effects of radiation can be estimated by reference to Figures 3 and 4. Radiation is an important mode of heat transfer in the first two inches of the gun tube where the Reynolds number is less than 10^6 . At higher values of the Reynolds number, convection is predominant. The effects of radiation are small because of the large value of the absorption coefficient at this pressure. Nearly all of the energy emitted by a volume of gas is absorbed by other gas volumes before it reaches the bore surface.

When the base of the projectile is 12.5 inches from the breech end, the conditions in the gun tube are

$$\begin{aligned} L &= 12.5 \text{ in} \\ P &= 12000 \text{ psi} \\ V_p &= 1750 \text{ ft/sec} \\ \rho &= 4.5 \text{ lb/ft}^3 \\ T &= 4000^\circ\text{R} \end{aligned}$$

These data were used to compute the values of Re_x and $u_\infty/k_R v$ given below.

<u>X (in)</u>	<u>Re_x</u>	<u>u_∞/k_R v</u>
0	0	0
1.0	9.61×10^5	2.34×10^2
2.0	3.84×10^6	4.70×10^2
3.0	8.64×10^6	7.06×10^2
4.0	1.54×10^7	9.45×10^2
5.0	2.40×10^7	1.17×10^3
6.0	3.46×10^7	1.41×10^3
7.0	4.71×10^7	1.65×10^3
8.0	6.15×10^7	1.88×10^3
9.0	7.79×10^7	2.12×10^3
10.0	9.61×10^7	2.36×10^3
11.0	1.17×10^8	2.58×10^3
12.0	1.38×10^8	2.82×10^3
12.5	1.50×10^8	2.94×10^3

The boundary layer thickness calculated from Equation 60 is 0.009 inch which is much less than that of the tube radius. The effects of radiation can again be estimated from Figures 3 and 4. Radiation is significant in the first three inches from the breech. Beyond this distance, turbulent convection is dominant.

In both of the examples given above, radiation was important only in the region near the breech. This will be true for all small caliber weapons because the propellant gas pressures are very high. At these high pressures, the gas is very dense and is nearly opaque to thermal radiation. Thus, while a large quantity of radiant energy is emitted by the propellant gas, most of this energy is absorbed before it strikes the bore surface. If the projectile velocity and the tube pressure were lower, radiation would be a more important mode of heat transfer. The absorption coefficient would be smaller, and more radiation would strike the bore surface. Also, the turbulent convective heat transfer would be less. Thus, radiation heat transfer would be relatively more important. These conditions may exist in some artillery weapons.

SUMMARY

Thermal radiation in gun tubes was analytically studied. A simplified physical model was used in this analysis. With this model, steady turbulent flow of an optically thick gray gas in the gun tube was considered. The flow in both undeveloped and fully developed turbulence was considered. The governing equations were derived and programmed for numerical evaluation. The numerical results were presented in dimensionless form so that they could be used to estimate the contribution of radiative heat transfer in any gun tube for which the flow conditions are known. The contribution of thermal radiation heat transfer in the XM140 gun tube was investigated, as an example. Radiation was important near the breech end. At locations of more than a few inches from the breech end, turbulent convection was the dominant mode of heat transfer. The primary reason that radiation is insignificant at distances further from the breech is that the absorption coefficient of the propellant gas is very large. While a significant amount of radiant energy is emitted by the gas, most of the radiant energy is absorbed before it strikes the bore surface. Radiation would be more important in guns operating at lower pressures and having lower projectile velocities.

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